Utilising Data and Knowledge for Policy Impact

A Case Study Using Bayesian Approach

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Rationale for New Estimates

- Potential for inaccurate results
- More new data generated
- Potential for better sample representation
- Importance of choosing a suitable method

Purpose & Context

- Purpose
  * To produce valid & credible estimate.
- Context
  * Previous attempts
  * Political sensitivity
  * Money/policies
  * Legal issues
  * Law enforcement
  * There are also data issues
  * Will be carefully scrutinised

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What is available?

DATA

- National Administrative Data
  * Large sample size
  * Intensive information
  * Lack of data entry validation
  * Considerable data-minning is required
  * Need to establish communication with the data vendor

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What is available?

Data

- National Administrative Data
  * Large sample size
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- National Survey:
  * Fill in the gaps
  * Provide useful information on unpaid travel time
  * Can't be linked directly to the administrative data

Knowledge

- Previous Estimates
  * Authoritative government organisations
  * Others from market research and small surveys
  * `Wide ranging` estimates based on different sample sizes

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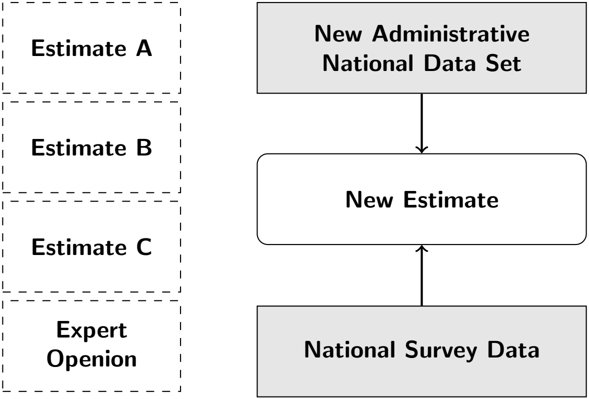
Knowledge

- Previous Estimates
  * Authoritative government organisations
  * Others from market research and small surveys
  * Wide ranging estimates based on different sample sizes
- Expert Opinion:
  * Different from previous estimates
  * Based on related research

Information summary



Accounting for prior knowledge

- Previous point estimates that varies
- All previous samples are drawn from same population
- Researcher's opinion based on observations
- We need full distribution estimate that account for all the above
- A Bayesian approach is most suitable for this job

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How can we make use of all the above?

Bayes' Theorem

$$P(A \mid B) = \frac{P(B \mid A)\, P(A)}{P(B)}$$

$$P(Parameter|Data) \propto P(Parameter) \times P(Data|Parameter)$$

Pros

- A formal approach for including prior knowledge in the analysis
- Flexibility in constructing appropriate model for the data
- Transparent, all modelling decisions are clear
- Full distributions instead of point estimates
- CI have more intuitive meaning

Cons

- Tailored estimation could be challenging to implement
- Computationally intensive
- Need defense of decisions

Hierarchical Bayesian Model

- A framework for capturing dependencies between parameters
- Treating all estimates as arising from a random process governed by hyperparameters
- More accurate than treating previous estimates as fixed prior
- Full posterior distributions for all parameters including previous estimates

Model specifications

- All estimates are based on samples drawn from same population

$$P(\theta_i|y_i) \propto P(y_i|\theta_i) \times P(\theta_i)$$

- θ_i are drawn from a distribution with unkown parameter vector ϕ
- ϕ is an unknown hyperparameter with its own posterior distribution

$$P(\theta_i, \phi|y_i) \propto P(y_i|\theta_i, \phi) \times P(\theta_i, \phi)$$

$$\underbrace{P(\theta, \alpha, \beta|y)}_{\text{Posterior}} \propto \underbrace{P(y|\theta)}_{\text{Likelihood}} \times \underbrace{P(\theta|\alpha, \beta)}_{\text{Prior}} \times \underbrace{P(\alpha, \beta)}_{\text{hyperprior}}$$

Model implementation

- All posterior densities could be derived, for example:

$$P(\theta_i|\alpha,\beta,y_i) \propto$$
$$(1-\theta_i)^{\beta+n_i-y_i-1}\prod_{i=1}^{I}\theta_i^{y_i}(1-\theta_i)^{n_i-y_i}\times$$
$$\prod_{i=1}^{I}\frac{\Gamma(\alpha+\beta)+n_i}{\Gamma(\alpha+y_i)\Gamma(\beta+n_i-y_i)}\theta_i^{\alpha+y_i-1}$$

- Stochastic integration via Markov Chain Monte Carlo
- A hyper Metropolis-Hasting/Gibbs sampling algorithm
- Estimated densities of θ_i, $\alpha/(\alpha+\beta)$
- R-Unix environment

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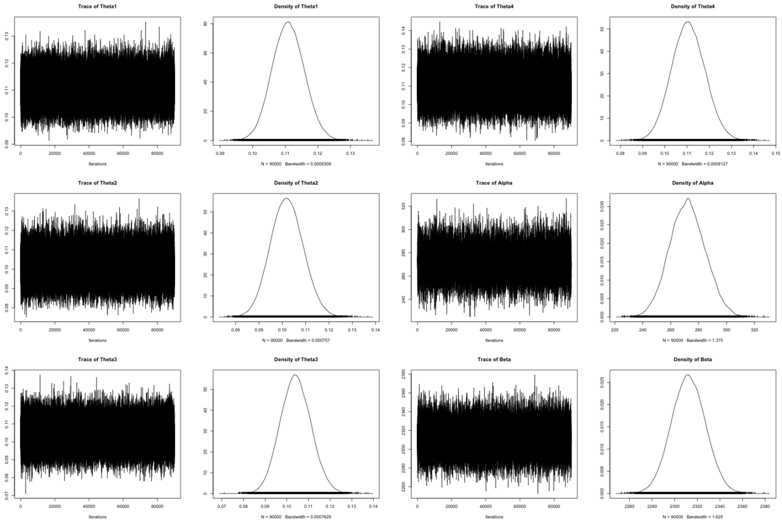
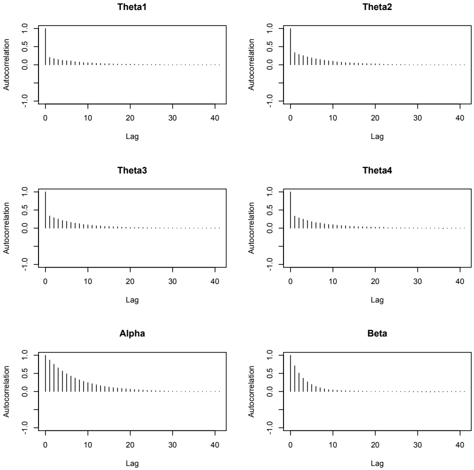
Output 1


Trace of Theta1
Density of Theta1
Trace of Theta4
Density of Theta4
Trace of Theta2
Density of Theta2
Trace of Alpha
Density of Alpha
Trace of Theta3
Density of Theta3
Trace of Beta
Density of Beta
Iterations
N = 90000 Bandwidth = 0.0005309
N = 90000 Bandwidth = 0.0008127
N = 90000 Bandwidth = 0.000757
N = 90000 Bandwidth = 1.375
N = 90000 Bandwidth = 0.0007629
N = 90000 Bandwidth = 1.625

Output 2


Theta1
Theta2
Theta3
Theta4
Alpha
Beta
Autocorrelation
Lag
-1.0 0.0 0.5 1.0
0 10 20 30 40

Communication & Interepretation

- Visualisation
- Communication process
- Providing tools for researchers to make decision

Reflection

- This is just an example
- Focusing on the suitability rather than convenience could be rewarding
- Recent developments offer new opportunities
  * New algorithms for stochastic integration
    * Sampling instead of numerical approximation
  * Choice of programming languages
    * C/C++, R, Stan
  * Significant hardware advancement
  * Availability of big data
- Skills Matrix
  * Subject matter expert
  * Mathematical statistics
  * Quantitative programming

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Further Reading

Adrian E. Raftery (2000)
Statistics in Sociology, 1950-2000
Journal of the American Statistical Association 95(450): 654 – 661.

Bruce Western (1999)
Bayesian Analysis for Sociologists
Sociological Methods & Research 28(1): 7 – 34

Bruce Western (2001)
Bayesian Thinking about Macrosociology
American Journal of Sociology 107(2): 353 – 378

Bob Carpenter, et all (2016)
Stan: A Probabilistic Programming Language
Journal of Statistical Software (in press)

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Thank You for Listening

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