The Significance of Statistical Methods in Utilising Data and Knowledge

Case Study Using a Hierarchical Bayesian Model

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August 29, 2014

Rationale for Presentation

- Funders requirements for evidence based research
- Data is getting more accessible
- More new data generated
- Different statistical techniques
- Potential for inaccurate results
- Importance of choosing a suitable method
- The following case study is an attempt to address the above

Purpose & Context

- Purpose
  - To produce valid & credible estimate.
- Context
  - Previous attempts
  - Political sensitivity
  - Money/policies
  - Legal issues
  - Law enforcement
  - There are also data issues
  - Will be carefully scrutinised

What is available?

Data

- National Administrative Data
  - Large sample size
  - Intensive information
  - Lack of data entry validation
  - Considerable data-minning is required
  - Need to establish communication with the data vendor

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- National Survey:
  - Fill in the gaps
  - Provide useful information on unpaid travel time
  - Can't be linked directly to the administrative data

Knowledge

- Previous Estimates
  * Authoritative government organisations
  * Others from market research and small surveys
  * `Wide ranging` estimates based on different sample sizes

Knowledge

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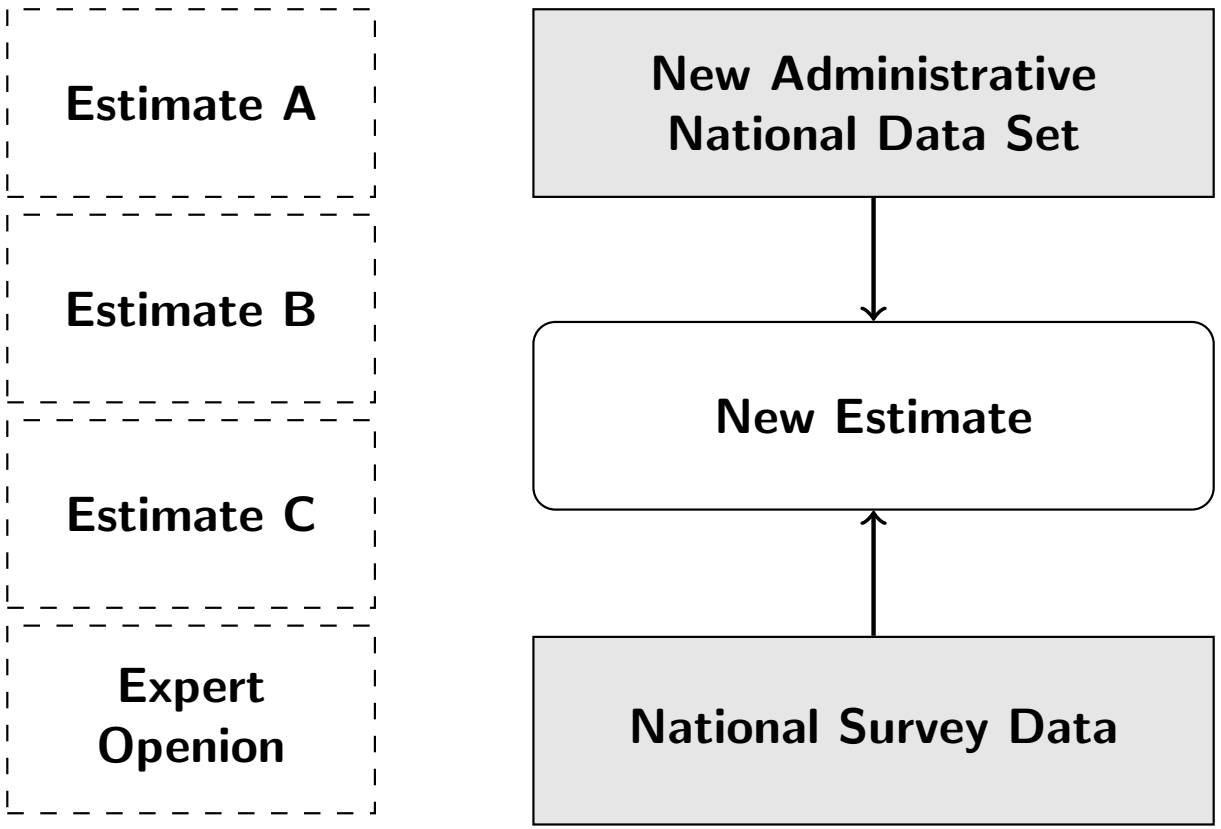
  - Authoritative government organisations
  - Others from market research and small surveys
  - Wide ranging estimates based on different sample sizes
- Expert Opinion:
  - Different from previous estimates
  - Based on related research

Information summary



Accounting for prior knowledge

- Previous point estimates that varies
- All previous samples are drawn from same population
- Researcher's opinion based on observations
- We need full distribution estimate that account for all the above
- A Bayesian approach is most suitable for this job

How can we make use of all the above?

Bayes' Theorem

$$P(A \mid B) = \frac{P(B \mid A)\, P(A)}{P(B)}$$

$$P(Parameter|Data) \propto P(Parameter) \times P(Data|Parameter)$$

Pros

- A formal approach for including prior knowledge in the analysis
- Flexibility in constructing appropriate model for the data
- Transparent, all modelling decisions are clear
- Full distributions instead of point estimates
- CI have more intuitive meaning

Cons

- Tailored estimation could be challenging to implement
- Computationally intensive
- Need defense of decisions

Hierarchical Bayesian Model

- A framework for capturing dependencies between parameters
- Treating all estimates as arising from a random process governed by hyperparameters
- More accurate than treating previous estimates as fixed prior
- Full posterior distributions for all parameters including previous estimates

Model specifications

- All estimates are based on samples drawn from same population

$$P(\theta_i|y_i) \propto P(y_i|\theta_i) \times P(\theta_i)$$

- θ_i are drawn from a distribution with unkown parameter vector ϕ
- ϕ is an unknown hyperparameter with its own posterior distribution

$$P(\theta_i, \phi|y_i) \propto P(y_i|\theta_i, \phi) \times P(\theta_i, \phi)$$

$$\underbrace{P(\theta, \alpha, \beta|y)}_{\text{Posterior}} \propto \underbrace{P(y|\theta)}_{\text{Likelihood}} \times \underbrace{P(\theta|\alpha, \beta)}_{\text{Prior}} \times \underbrace{P(\alpha, \beta)}_{\text{hyperprior}}$$

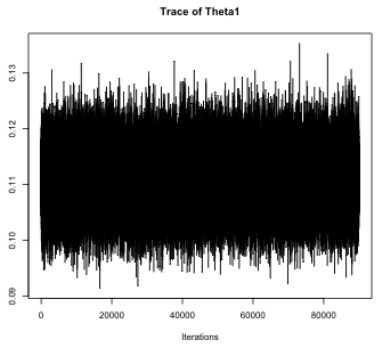
Model implementation

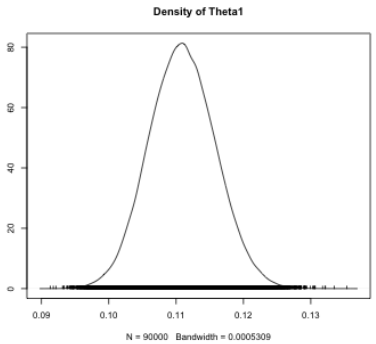
- All posterior densities could be derived, for example:

$$P(\theta_i|\alpha,\beta,y_i) \propto$$
$$(1-\theta_i)^{\beta+n_i-y_i-1}\prod_{i=1}^{I}\theta_i^{y_i}(1-\theta_i)^{n_i-y_i}\times$$
$$\prod_{i=1}^{I}\frac{\Gamma(\alpha+\beta)+n_i}{\Gamma(\alpha+y_i)\Gamma(\beta+n_i-y_i)}\theta_i^{\alpha+y_i-1}$$

- Stochastic integration via Markov Chain Monte Carlo
- A hyper Metropolis-Hasting/Gibbs sampling algorithm
- Estimated densities of θ_i, $\alpha/(\alpha+\beta)$
- R-Unix environment

Output 1


Trace of Theta1
Iterations


Density of Theta1
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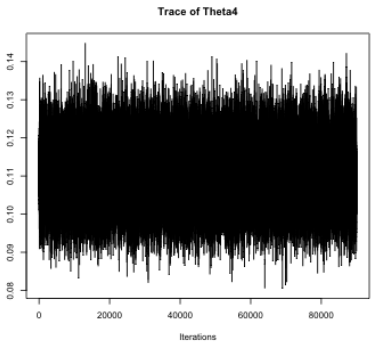
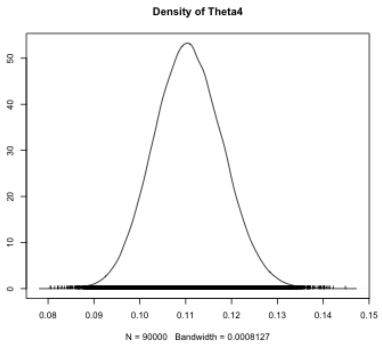
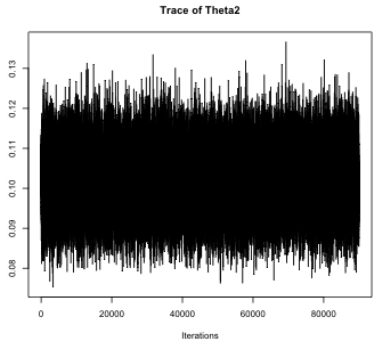
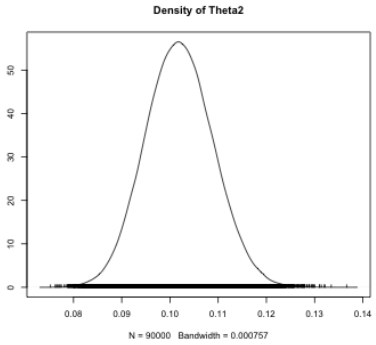

Trace of Theta4
Iterations


Density of Theta4
N = 90000 Bandwidth = 0.0008127


Trace of Theta2
Iterations


Density of Theta2
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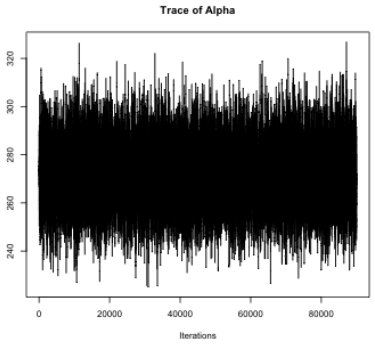
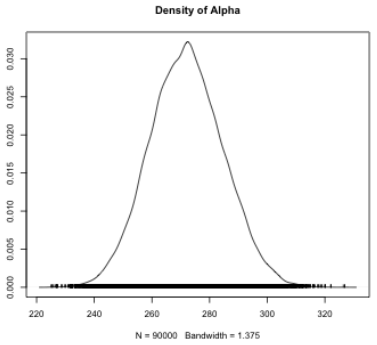
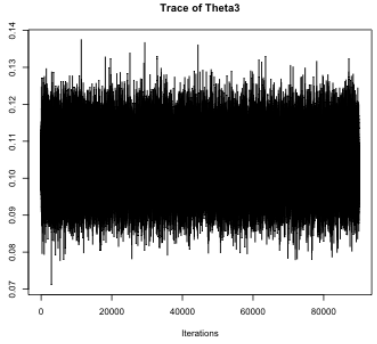
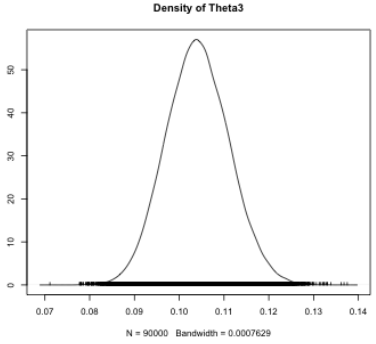

Trace of Alpha
Iterations


Density of Alpha
N = 90000 Bandwidth = 1.375


Trace of Theta3
Iterations


Density of Theta3
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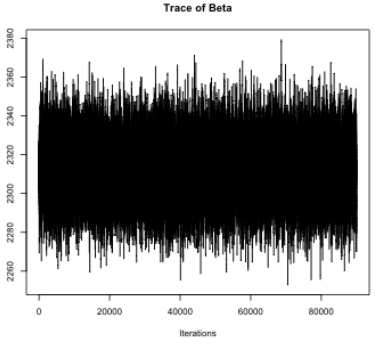
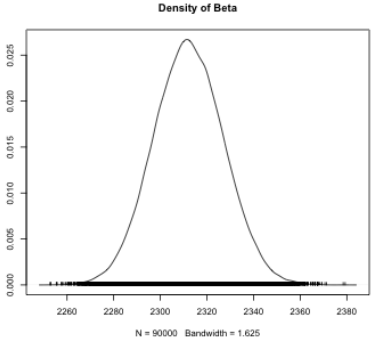
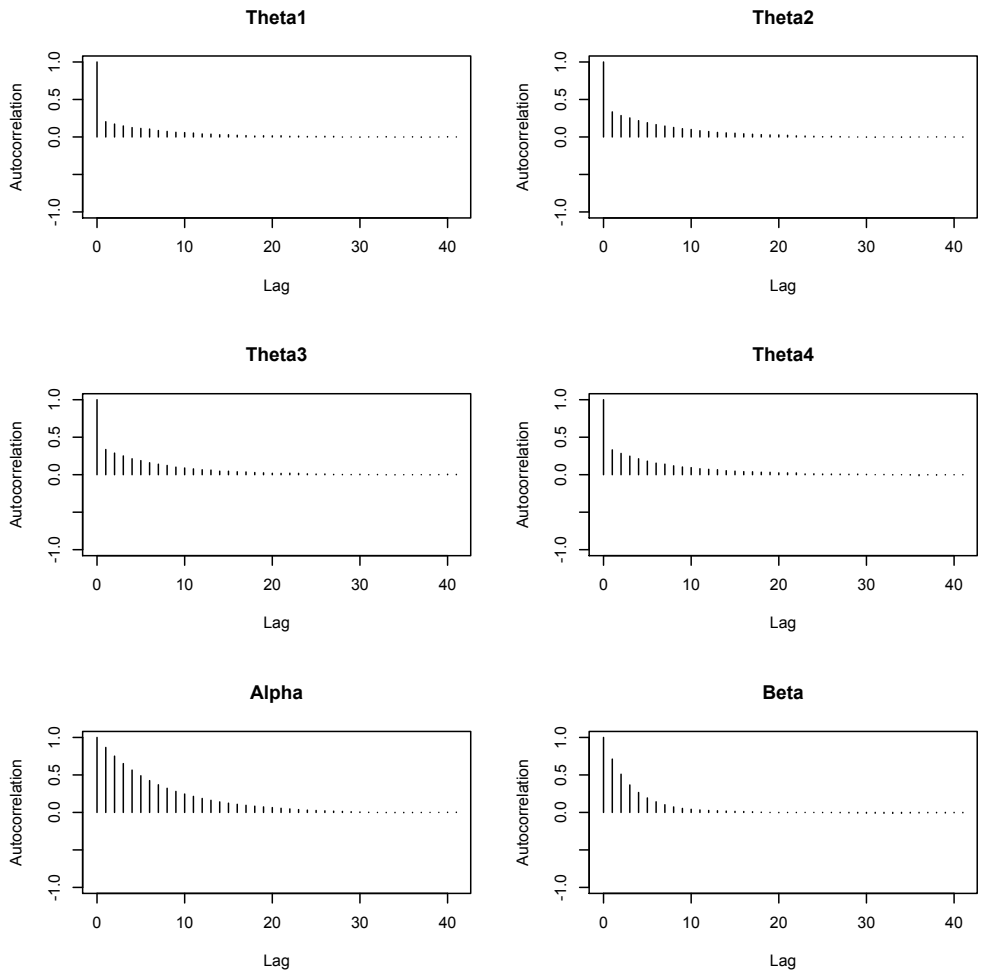

Trace of Beta
Iterations


Density of Beta
N = 90000 Bandwidth = 1.625

Output 2



Communication & Interepretation

- Visualisation
- Communication process
- Providing tools for researchers to make decision

Reflection

- This is just an example
- Focusing on the suitability rather than convenience could be rewarding
- Recent developments offer new opportunities
  - New algorithms for stochastic integration
    - Sampling instead of numerical approximation
  - Choice of programming languages
    - C/C++, R
  - Significant hardware advancement
  - Availability of big data
- Skills Matrix
  - Subject matter expert
  - Mathematical statistics
  - Quantitative programming

Further Reading

Adrian E. Raftery (2000)

Statistics in Sociology, 1950-2000

Journal of the American Statistical Association 95(450): 654 – 661.

Bruce Western (1999)

Bayesian Analysis for Sociologists

Sociological Methods & Research 28(1): 7 – 34

Bruce Western (2001)

Bayesian Thinking about Macrosociology

American Journal of Sociology 107(2): 353 – 378

Thank You for Listening

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